Exponential peak and scaling of work fluctuations in modulated systems

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We show that steady-state work fluctuations in periodically modulated systems display universal features, which are not described by the standard fluctuation theorems. Modulated systems often have coexisting stable periodic states. We find that work fluctuations sharply increase near a kinetic phase transition where the state populations are close to each other. We also show that the work variance displays scaling with the distance to a bifurcation point where a stable state disappears and find the critical exponent for a saddle-node bifurcation.

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Since their discovery in the early 1990s [1–3], fluctuation theorems have been attracting increasing interest. Their goal is to establish general features of fluctuating systems away from thermal equilibrium. A major "test bed" for fluctuation theorems is provided by dynamical systems with a few degrees of freedom coupled to a thermal bath, a Brownian particle being an example. Much of the theoretical and experimental work refers to (i) modulated linear systems, where fluctuations have been studied both in transient and stationary regimes [4–8] and (ii) pulse-driven nonlinear systems, initially at thermal equilibrium [9–13].

Fluctuations in nonequilibrium dynamical systems have been attracting much attention also in a different context. They play an important role in mesoscopic vibrational systems of current interest, including Josephson junctions, nano- and micromechanical resonators, and trapped electrons and atoms. Because damping of the vibrations is typically weak, even a moderately strong resonant force can excite them to comparatively large amplitudes, where the nonlinearity becomes substantial. As a result, the system may have two or more coexisting stable states of forced vibrations [14]. Fluctuations can cause switching between these states [15] and thus significantly affect the system even where they are small on average. Many features of switching in mesoscopic systems and a broad range of switching related phenomena and applications, from resonant frequency mixing to high-frequency stochastic resonance and to quantum measurements have been recently studied experimentally [16–24] and theoretically [25–28].

In this paper we provide a general analysis of work fluctuations in periodically modulated nonlinear dynamical systems coupled to a bath. The results are applied to a model that describes the systems [16–23]. We find the distribution of fluctuations of work done on a nonlinear system by the modulating force over a long time τ.

In common with systems close to thermal equilibrium, the work variance σ^2 is proportional to the average work $\langle W\rangle$. However, in contrast to equilibrium systems and to modulated linear systems, in nonlinear systems the proportionality coefficient is not universal. Nevertheless, it has universal features. They emerge near critical points. In particular, the variance becomes exponentially large in bistable systems in the range of a kinetic phase transition where stationary populations of the vibrational states are close to each other. This parameter range has similarity with the region of a first-order phase transition where molar fractions of the coexisting phases are close to each other [15,29].

The power absorbed from the force in different vibrational states $i=1,2$ is different, generally. Therefore switching back and forth between the states leads to large power fluctuations. Their correlation time is determined by the switching rates. For a small characteristic intensity D of the noise that comes from the bath, these rates are small compared to the dynamical relaxation rate in the absence of noise t_r^{-1} and the modulation frequency ω_F. Then of primary interest are period-averaged switching rates ν_{ij}. They often display activation dependence on D, with $\nu_{ij}\propto\exp(-R_i/D)$, where R_i is the characteristic activation energy of a transition $i\to j$. Since work fluctuations accumulate power fluctuations, and the typical accumulation time for interstate fluctuations is $\sim\nu_{ij}^{-1}$, the exponential smallness of ν_{ij} may lead to an exponentially large factor in the work variance σ^2.

We consider a fairly general model, a nonlinear classical dynamical system modulated by a periodic force $F(t)=\Sigma_n\tilde{F}(n)\exp(in\omega_F t)$; the coupling energy is $-F(t)q$, where q is the system coordinate. The system is additionally coupled to a bath, which leads to relaxation and fluctuations. Work done by the force over time τ is

$$W\equiv W(\tau)=\int_0^\tau dtF(t)\dot{q}(t). \quad (1)$$

We are interested primarily in steady-state fluctuations, i.e., we assume that the system had come to the steady state well before the time $t=0$ when the work (1) started to be measured. This steady state is periodic in time with modulation period $\tau_F=2\pi/\omega_F$. We further assume that the time τ largely exceeds the characteristic decay time of correlations in the system $t_{\rm corr}$. Often for bistable systems $t_{\rm corr}\sim 1/\nu_{ij}$. Because we are interested in the large-τ limit, the results also apply if work is defined as $W=-\int_0^\tau dt\dot{F}(t)q(t)$; they can be easily generalized to a coordinate-dependent force.

Work fluctuations can be expressed in terms of the correlation function of velocity fluctuations $Q(t,t')=\langle\delta\dot{q}(t)\delta\dot{q}(t')\rangle$, where $\langle\cdots\rangle$ means ensemble average and $\delta\dot{q}(t)=\dot{q}(t)-\langle\dot{q}(t)\rangle$. Because the system is in a steady periodic state, we have $Q(t,t')=Q(t+\tau_F,t'+\tau_F)$, and therefore

$$Q(t,t')=\sum_n Q(n;t-t')\exp[in\omega_F(t+t')/2]. \quad (2)$$

We first consider the variance of the work distribution $\sigma^2 \equiv \sigma^2(\tau) = \langle(\delta W)^2\rangle$, where $\delta W = W(\tau) - \langle W(\tau)\rangle$. In the limit of large τ

$$\sigma^2 \approx 2\pi\tau \sum_{n,m} \tilde{F}(n)\tilde{F}^*(m)\tilde{Q}\left(m-n; \frac{n+m}{2}\omega_F\right),$$

$$\tilde{Q}(n;\omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} dt e^{i\omega t} Q(n;t). \qquad (3)$$

Here we have taken into account that the correlation functions $Q(n;t-t')$ decay on time $t_{\rm corr}$ much smaller than τ. Therefore the limits of integration over $t-t'$ could be extended from $-\infty$ to ∞.

Decay of correlations on a time scale small compared to τ allows one to simplify the expressions for higher-order moments of δW in a standard way. The third moment $\langle(\delta W)^3\rangle$ is determined by the integral over t_1, t_2, t_3 of the appropriately weighted correlator $\langle \delta\dot{q}(t_1)\delta\dot{q}(t_2)\delta\dot{q}(t_3)\rangle$. Because $|t_1 - t_2|, |t_1 - t_3| \lesssim t_{\rm corr}$, we have $\langle(\delta W)^3\rangle \propto \tau$ for large τ, and therefore $\langle(\delta W)^3\rangle / \langle(\delta W)^2\rangle^{3/2} \propto \tau^{-1/2}$, i.e., the third moment is small for large τ. The fourth moment $\langle(\delta W)^4\rangle$ is determined by the integral of the correlator $\langle \delta\dot{q}(t_1)\delta\dot{q}(t_2)\delta\dot{q}(t_3)\delta\dot{q}(t_4)\rangle$. The main contribution to this integral comes from decoupling the correlator into pairs $\langle \delta\dot{q}(t_{n_1})\delta\dot{q}(t_{n_2})\rangle\langle \delta\dot{q}(t_{n_3})\delta\dot{q}(t_{n_4})\rangle$ with $|t_{n_1} - t_{n_2}|, |t_{n_3} - t_{n_4}| \lesssim t_{\rm corr}$ while $|t_{n_1} - t_{n_3}| \sim \tau$ $(n_i = 1, \ldots, 4)$. This gives $\langle(\delta W)^4\rangle \approx 3\langle(\delta W)^2\rangle^2 \propto \tau^2$. Higher-order correlations in $\langle(\delta W)^4\rangle$ give a comparatively small contribution $\propto \tau$. The analysis can be immediately extended to higher moments of δW. It shows that the overall distribution of work fluctuations $P(W)$ is Gaussian,

$$P(W) = (2\pi\sigma^2)^{-1/2} \exp[-(W - \langle W\rangle)^2/2\sigma^2]. \qquad (4)$$

It follows from Eqs. (3) and (4) that $P(W)/P(-W) = \exp(2W\langle W\rangle/\sigma^2)$, as in the stationary state fluctuation theorem for systems close to thermal equilibrium and for modulated linear systems, and the variance of the work distribution $\sigma^2 \propto \tau \propto \langle W\rangle$. However, for strong periodic modulation there is no known general expression that would relate the average velocity $\langle \dot{q}(t)\rangle$ to the modulating force in terms of the correlation functions $\tilde{Q}(n;\omega)$ and thus give $\sigma^2/\langle W\rangle$. Moreover, as we show, the ratio $\sigma^2/\langle W\rangle$ may display sharp narrow peaks as a function of system parameters.

We now consider a system with two stable periodic states $j = 1, 2$. For weak noise, it mostly performs small fluctuations about the stable states and only occasionally switches between them. Then, to leading order in the noise intensity D the average work is a sum of partial works $W_{1,2}$ in each of the states weighted with the stationary populations of the states $w_{1,2}^{\rm st}$ [15],

$$\langle W\rangle = \sum_{j=1,2} w_j^{\rm st} W_j, \quad W_j = \omega_F \tau \sum_n in\tilde{F}^*(n)\tilde{q}_j(n),$$

$$w_1^{\rm st} = \nu_{21}/(\nu_{12} + \nu_{21}), \quad w_2^{\rm st} = 1 - w_1^{\rm st}. \qquad (5)$$

Here, $\tilde{q}_j(n)$ is the Fourier component of the coordinate $q_j(t)$ in a state j, $q_j(t) = \Sigma_n \tilde{q}_j(n)\exp(in\omega_F t)$.

In contrast to the average work, the variance σ^2 has contributions of two different types. One comes from small-amplitude fluctuations about the stable states. It is given by the sum of partial variances $\sigma_{1,2}^2$ weighted with the state populations. The variances $\sigma_{1,2}^2$ can be obtained by linearizing equations of motion about the corresponding stable state and can be written as

$$\sigma_j^2 = C_j D W_j, \quad j = 1, 2. \qquad (6)$$

The constants $C_{1,2}$ depend on the system dynamics. For a system coupled to a thermal bath at temperature T, in the weak-field limit we have $\sigma_j^2 = 2k_B T W_j$. However, for strong field this relation does not hold in nonlinear systems, generally.

Fluctuations about q_j become large near a saddle-node bifurcation point where the state j disappears. Here, one of the motions of the system is slow [30], there emerges a "soft mode." Let $q_j^{(c)}(t)$ denote the bifurcational (critical) position of the state j. Close to it one can quite generally write $q(t) - q_j^{(c)}(t) = q_{\rm sm}(t)\varkappa_j(t)$, where $\varkappa_j(t) = \varkappa_j(t + \tau_F)$ is a periodic function, whereas $q_{\rm sm}(t)$ is a slowly varying amplitude that depends on initial conditions. Upon rescaling, the equation of motion for $q_{\rm sm}(t)$ can be written as

$$\dot{q}_{\rm sm} = q_{\rm sm}^2 - \eta + f(t), \quad \langle f(t)f(t')\rangle = 2D'\delta(t - t'). \qquad (7)$$

Here, η is the distance to the bifurcation point, for example, the scaled difference between the amplitude of the field and its value at the bifurcation point. The noise $f(t)$ can be quite generally assumed to be white because of the slowness of $q_{\rm sm}(t)$; its intensity is $D' \propto D$.

For $\eta > 0$ in the absence of noise the system (7) has a stable state where $q_{\rm sm} = -\eta^{1/2}$. Small fluctuations about this state have variance $D'\eta^{-1/2}/2$ and decay over time $t_r = \eta^{-1/2}/2$. Therefore, from Eq. (3), near the bifurcation point where a state j disappears

$$\sigma_j^2/W_j = \tilde{C}_j D/\eta. \qquad (8)$$

Factor $\tilde{C}_j$ is independent of D and η, and W_j does not diverge for $\eta \to 0$.

Equation (8) shows that the partial work variance scales as η^{ξ} with the distance to the bifurcation point. The critical exponent is $\xi = -1$.

The other contribution to σ^2 comes from fluctuation-induced interstate transitions. The transitions lead to fluctuations of the state populations $w_j(t)$. These fluctuations are slow

$$\langle \delta w_1(t)\delta w_1(t')\rangle = w_1^{\rm st} w_2^{\rm st} \exp[-\nu|t - t'|],$$

$$\nu = \nu_{12} + \nu_{21}, \qquad (9)$$

where $\delta w_1(t) = w_1(t) - w_1^{\rm st} = -\delta w_2(t)$. In turn, they lead to slow fluctuations of the velocity $\dot{q}(t) \approx \Sigma_j \dot{q}_j(t) w_j(t)$ with decay time given by the reciprocal total switching rate $\nu^{-1} \gg t_{\rm r}, \tau_F$.

From Eqs. (2), (3), (5), (9), the contribution to the work variance from interstate switching is

$$\sigma_{\rm sw}^2 \approx \mathcal{M}(\nu\tau)^{-1}(W_1 - W_2)^2, \quad \mathcal{M} = 2w_1^{\rm st}w_2^{\rm st}, \qquad (10)$$

and the total variance is

$$\sigma^2 = \sum_j w_j^{\rm st}\sigma_j^2 + \sigma_{\rm sw}^2. \qquad (11)$$

Equation (10) is one of the central results of the paper. It shows that the switching-induced contribution to the work variance is proportional to the squared difference of the partial works in the stable states and is inversely proportional to the switching rate ν. The rate $\nu \propto \exp(-\min_i R_i/D)$ is exponentially small for small noise intensity. Respectively, the variance (10) can be exponentially large compared to the variance due to small fluctuations about attractors (6).

Factor $\mathcal{M}$ in Eq. (10) sharply depends on the parameters of the system and the field $F(t)$. It is small,

$$\mathcal{M} \propto \exp[-|R_1 - R_2|/D], \qquad (12)$$

except for a narrow range of the kinetic phase transition where the switching activation energies are close to each other, $|R_1 - R_2| \lesssim D$. At its maximum $\mathcal{M}=1/2$. Equations (10) and (12) show that the ratio $\sigma^2/\langle W\rangle \propto \mathcal{M}\nu^{-1}$ displays an exponentially sharp peak at the kinetic phase transition. We note that factor $\mathcal{M}$ determines also the intensity of very narrow peaks (of width $\nu \ll t_{\rm r}^{-1}$) in the power spectra of modulated bistable systems and the spectra of absorption/amplification of an additional field [15,31]. Its exponential dependence on the distance to the kinetic phase transition was seen in simulations [32] and experiment [22].

The above formulation can be easily generalized to multidimensional systems with the energy of coupling to the force of the form $-\mathbf{q}\cdot\mathbf{F}(t)$. Both Eq. (10) for $\sigma_{\rm sw}^2$ and Eq. (8) for scaling of σ_j^2 near a saddle-node bifurcation point hold in this case as well.

We now illustrate the results using as an example a resonantly driven underdamped Duffing oscillator, a model that applies to a number of recent experiments on Josephson junctions and nanomechanical and micromechanical resonators. In the absence of noise the oscillator dynamics is described by the equation

$$\ddot{q} + \omega_0^2 q + \gamma q^3 + 2\Gamma\dot{q} = A\cos\omega_F t. \qquad (13)$$

We assume that the detuning of the field frequency from the oscillator eigenfrequency $\delta\omega=\omega_F-\omega_0$ and the friction coefficient Γ are small: $|\delta\omega|, \Gamma \ll \omega_0$. Then the oscillator can display bistability of forced vibrations already for a comparatively small driving amplitude A, where the vibrations remain almost sinusoidal, $q_j(t)\approx a_j\cos(\omega_F t+\phi_j)$ $(j=1,2)$. Explicit expressions for the amplitudes $a_{1,2}$ and phases $\phi_{1,2}$ are known [14] and the interstate switching rates are well understood [15,17,19,20,32].

The partial work in a stable vibrational state j is $W_j=\Gamma\tau\omega_F^2 a_j^2$. The partial variances σ_j^2 due to thermal noise can be calculated using the approach [15,32]

$$\frac{\sigma_j^2}{W_j} = 2k_BTZ_j^{-2}[(Z_j-2)^2 + 4\Omega^2(Y_j-1)^2],$$

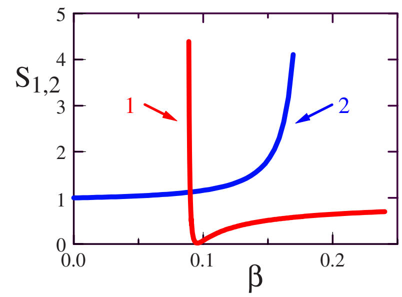

FIG. 1. (Color online) Scaled ratios of the partial work variance to mean partial work $S_j=\sigma_j^2/2k_BTW_j$, $j=1,2$, as functions of the reduced squared modulation amplitude β for a Duffing oscillator; $\Omega^{-1}=\Gamma/\delta\omega=0.3$. The curves 1 and 2 refer to the branches of large- and small-amplitude vibrations, respectively. Functions $S_{1,2}$ diverge at the bifurcation points where the corresponding branches disappear, $\beta_B^{(1)}\approx 0.088$ and $\beta_B^{(2)}\approx 0.18$.

$$\Omega = \delta\omega/\Gamma, \quad Z_j = 1 + \Omega^2(Y_j - 1)(3Y_j - 1), \qquad (14)$$

where $Y_j=3\gamma a_j^2/8\omega_F\delta\omega$. Equation (14) refers to the case where fluctuations and friction come from coupling to the same thermal reservoir; in fact, it is not limited to the model (13) and applies in a general case where the density of states of the reservoir weighted with the interaction is smooth near ω_F.

It follows from Eq. (14) that in the linear-response regime, where $Y_j \propto A^2$, $Y_j \ll 1$, the ratio $\sigma_j^2/W_j \to 2k_BT$ is given by the standard stationary state work fluctuation theorem. This is in agreement with the recent theoretical and experimental results on a periodically modulated linear system [7].

However, for stronger driving the ratio (14) is no longer given by $2k_BT$. When the field amplitude A or frequency ω_F approach their bifurcational value, we have

$$\sigma_j^2/W_j \approx 2k_BTG_j(\Omega)\eta^{-1}, \quad \eta = \beta - \beta_B^{(j)}(\Omega). \qquad (15)$$

Here, $\beta=3\gamma A^2/32\omega_F^3(\delta\omega)^3$ is the reduced field amplitude, $\beta_B^{(1,2)}$ are the bifurcational values of β [14], and $G_j=\Omega^{-2}\beta_B^{(j)}/Y_{jB}(3Y_{jB}-2)$ (Y_{jB} is the bifurcational value of Y_j). In agreement with Eq. (8), σ_j^2/W_j scales as η^{-1} with the distance η to the bifurcation point. The full dependence of σ_j^2/W_j on β is shown in Fig. 1.

It should be possible to see the scaling (15) in modulated mesoscopic oscillators for the same conditions in which the scaling of the switching activation energies was seen [19,20]. Similarly, the exponential peak of the work variance should be seen in experiments analogous to those where there were observed other kinetic phase transition phenomena such as super narrow peaks in the power spectra, high-frequency stochastic resonance, and fluctuation-enhanced frequency mixing [22,24].

Experiments on scaling and on the kinetic phase transition should be conducted in a different way. Scaling can be observed in the quasistationary regime. The system should be prepared in the metastable state near a bifurcation point, and the duration of the experiment should be shorter than the lifetime of the state. In contrast, the exponential peak of the work variance can be seen provided the duration of the measurement exceeds the reciprocal switching rate ν^{-1}.

In conclusion, we have considered work fluctuations for nonlinear systems modulated by a strong periodic field. We demonstrate that the standard steady-state work fluctuation theorem does not apply to nonlinear systems, generally. Nevertheless, work fluctuations may display system-independent features. If a system has coexisting stable vibrational states, the ratio of the work variance to the average work is proportional to the reciprocal rate of interstate switching. It has a sharp exponentially high peak as a function of the distance to the kinetic phase transition. Near a saddle-node bifurcation point where one of the vibrational states disappears, in the quasistationary regime the work variance displays scaling dependence on the distance to the bifurcation point. The results apply to a broad range of vibrational systems of current interest, from trapped electrons to Josephson junctions and to nanomechanical and micromechanical resonators.

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